STRUCTURAL DESIGN OF TRANSVERSE RING FRAMES

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Submitted to the Department of Naval Architecture and Marine Engineering on 18 May 1967, in partial fulfillment of the requirements for the degree of Naval Engineer and the degree of Master of Science in Naval Architecture and Marine Engineer.

ABSTRACT

The relation of the total structure weight of a two-dimensional ring frame to the stress distribution in each of the elements is investigated. The total weight of the two-dimensional structures investigated were a minimum when design stresses had been attained in all elements of the structure.

An expression is developed that approximates the change in stress distribution in a member when additions or deletions of material are made in the member itself and its associated members.

This expression is used as the basis for a rational design technique that converges to a ring frame of minimum total weight.

Thesis Supervisor: J. Harvey Evans
Title: Professor of Naval Architecture

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NOTATION

A	Cross Sectional Area
FEM	Fixed End Moment
I	Moment of Inertia
k	Distribution Factor $= \dfrac{I_i/L_i}{\sum_n I/L}$
K	Matrix of Member Properties
K'	Matrix of Updated Member Properties
K''	Matrix of Final Member Properties
L	Length
M_E	End Moment
M_F	Field Moment
N	Axial Force
V	Shear Force
W	External Load
Z	Section Modulus
σ	Stress
ΔM	Moment Change
Δk	Change in Distribution Factor

I. INTRODUCTION

The transverse structure of a wall sided ship consists primarily of bottom and deck beams, side frames and stanchions. This structure is called the transverse ring frame. The loads this structure is subjected to are side and bottom water pressure and the ship weights as well as dynamic loadings.

The structural analysis of this type of structure, in two and three dimensions, is discussed in considerable detail in the literature. Computer solutions for both the two and three dimensional aspects are available. These rapid analysis tools are necessary prerequisites for a rational design synthesis.

In its most general form the optimization of the transverse structure entails the specification of the transverse frame spacing, the effect of longitudinal frame spacing, and the distribution of material throughout the frame. This thesis deals with the last part of the process, the optimium distribution of material throughout the ring frame of a wall sided ship once the transverse and longitudinal frame spacings are fixed. The optimum structure is defined as the least weight combination of deck beam, side frames, etc., that will carry the design loads.

II. MINIMUM WEIGHT STRUCTURES

The end and field moments that a loaded beam element develops are directly related to its end restraint. These moment distributions are subject to wide variations. The bending moment diagrams for a beam element supporting a distributed load are reproduced in Figure 1.

Full rotational restraint at both ends produces a symmetrical moment distribution and one in which the maximum moments occur at the ends. The field moment dominates when the moment restraint at the ends is zero. The intermediate restraint case is characterized by a symmetrical moment distribution and identical end and field moments. In every case, limiting or design stresses are reached only at discrete points in the element.

The size or weight of any element depends on the maximum moment developed when the design load is applied. The wide variation in maximum moments is accompanied by corresponding variations in the weight per unit length of members. Table 1 lists, for the ideal section developed in Appendix C, the variation in weight for the structural configurations of Figure 1. The absolute minimum weight per unit length occurs when the maximum stress occurs at three points simultaneously in the beam element. The specific weight increases by thirty percent going from the fixed-fixed beam to the free-free or fixed-free beam. Doubling the maximum moment for this ideal section increases the section weight by sixty percent.

The weight ratios of Table 1 underline the fact that the total weight of any structural system is related to how the individual elements interact. A minimum total weight of structure can be obtained by substituting members whose moduli satisfy the intermediate restraint condition. In most cases, the actual moment distributions that obtain when the structure is assembled will not be identical to those assumed conditions. The final total structural weight will depend on how the material distribution has been altered to obtain convergence. This, in turn, depends on the design criteria. The examples that follow illustrate how the total weight varies when the design criteria is that design stress must exist in at least one element; the stress in all other members must be equal to or less than the design stress.

The first example, Figure 2, contains only elements subjected to bending. The length to depth ratio of each element is assumed to be large enough to exclude shear effects.

An initial choice of members based upon the intermediate restraint condition and whether the elements carry an external load gives:

$$Z_2 = \frac{1/16\ W_2\ L_2^2}{\sigma_{Design}}; \quad Z_1 = 0 = Z_3$$

The structure made up of these members does not possess adequate strength. The maximum stress in member two is greater than the design stress. The minimum weight members do not form an acceptable structure.

This structure can only be brought up to an acceptable level of strength by adding material. The weight of the final structure satisfying the design criteria will vary considerably depending on where the material is added. There are two directions available. One is to add material so that the moments at the nodes are very small. This will cause the field moment in member two to predominate. The other members will be very small. The second approach is to increase the size of all the members. This will bring about redundant moments at the nodes which are substantial.

The weight curves of Figure 3 illustrate the trade off possibilities that exist when all the members are varied. Large increases in the specific weight of members one and three are accompanied by relatively small increases in member one as the distribution factor approaches one. These effects are summarized in the total weight curve. These curves, for all the cases looked at, have definite minimums. When $L_2 = 5L_1 = 5L_3$, the curve shown in Figure 3, the minimum total weight occurred when all member sizes were equal and the node moments were maximum. At this minimum point, design stresses were obtained in all members.

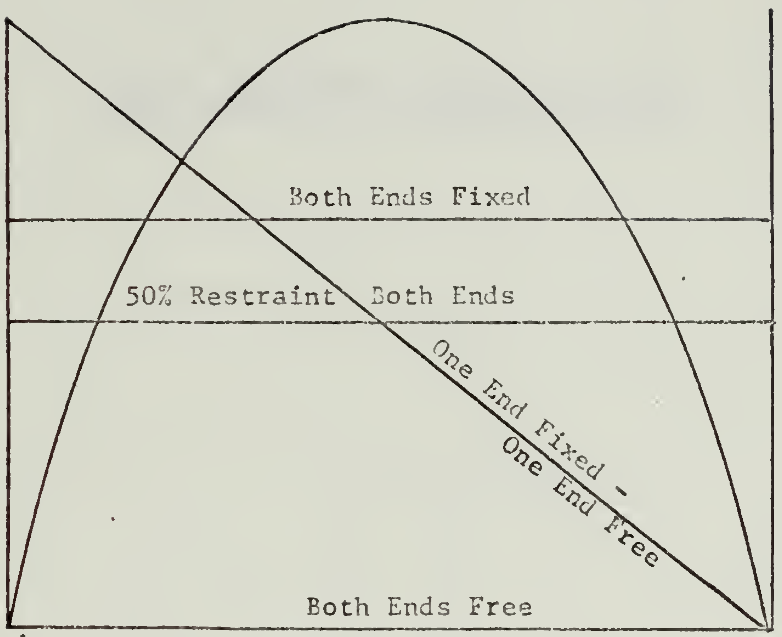
The following example is identical to the preceding except that all members have to support an external load. The loading is such that the fixed end moment of the middle span is twice that of the adjacent members; load symmetry is maintained.

As before, the structure made up of least weight members, corresponding to the intermediate restraint condition, is not satisfactory. All members are overstressed. Sufficient strength can only be obtained by adding material.

Total weight curves for this structure are shown in Figure 4. Each of these curves have minimum points which occur at $k = .5$. At this point, all members are of equal size and the node moments govern. It is obvious then that the minimum total weight occurs when design stress levels exist in all the members. At all other values for the distribution factor, one or two members have too much material.

Only bending stresses were considered in these examples. The substitution of a vertical member at each of the supports requires that direct stresses be accounted for (see Appendix B). The total weight curve for this structure also has a unique minimum. This minimum occurs when design stresses exist in all members.

The iteration technique developed in this thesis converges on a structure which, when the design load is applied, develops design stresses in each member simultaneously. These stresses may develop at the ends of the members or at any point along the members. The structures which this design process converges to represents the optimum distribution of material for the given loads and the fixed frame spacing. The total weight of these structures is a minimum. Any other combination of members will result in a higher total weight of structure; the stress in some or all the members will be less than the design stress.



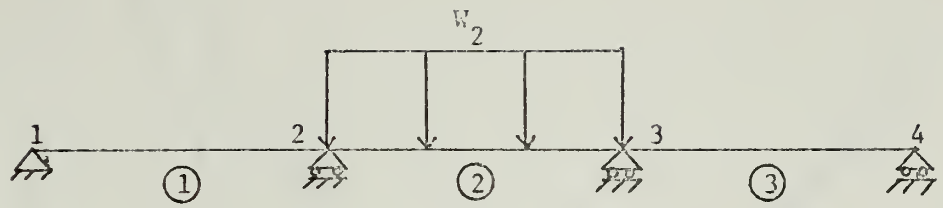
MOMENT DIAGRAMS FOR A BEAM SUPPORTING A DISTRIBUTED LOAD

FIGURE 1

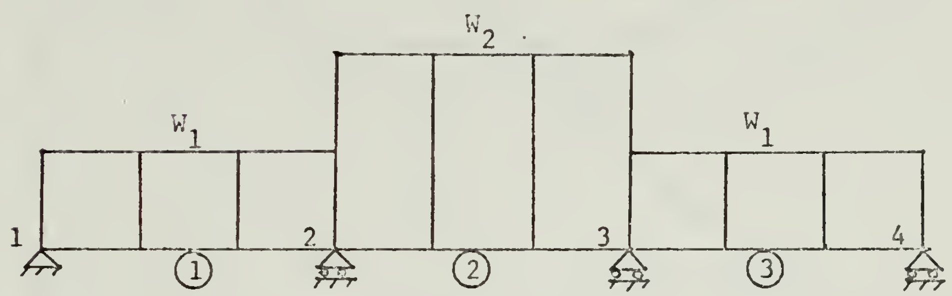
TABLE 1

Maximum Moment	Position	Moment Ratio	Weight Ratio
$1/16\ WL^2$	End, Field	M_o	W_o
$1/12\ WL^2$	End	$1.33\ M_o$	$1.21\ W_o$
$1/8\ WL^2$	End or Field	$2.0\ M_o$	$1.59\ W_o$

WEIGHT AND MOMENT RELATIONS FOR THE
BEAM SUPPORTING A DISTRIBUTED LOAD



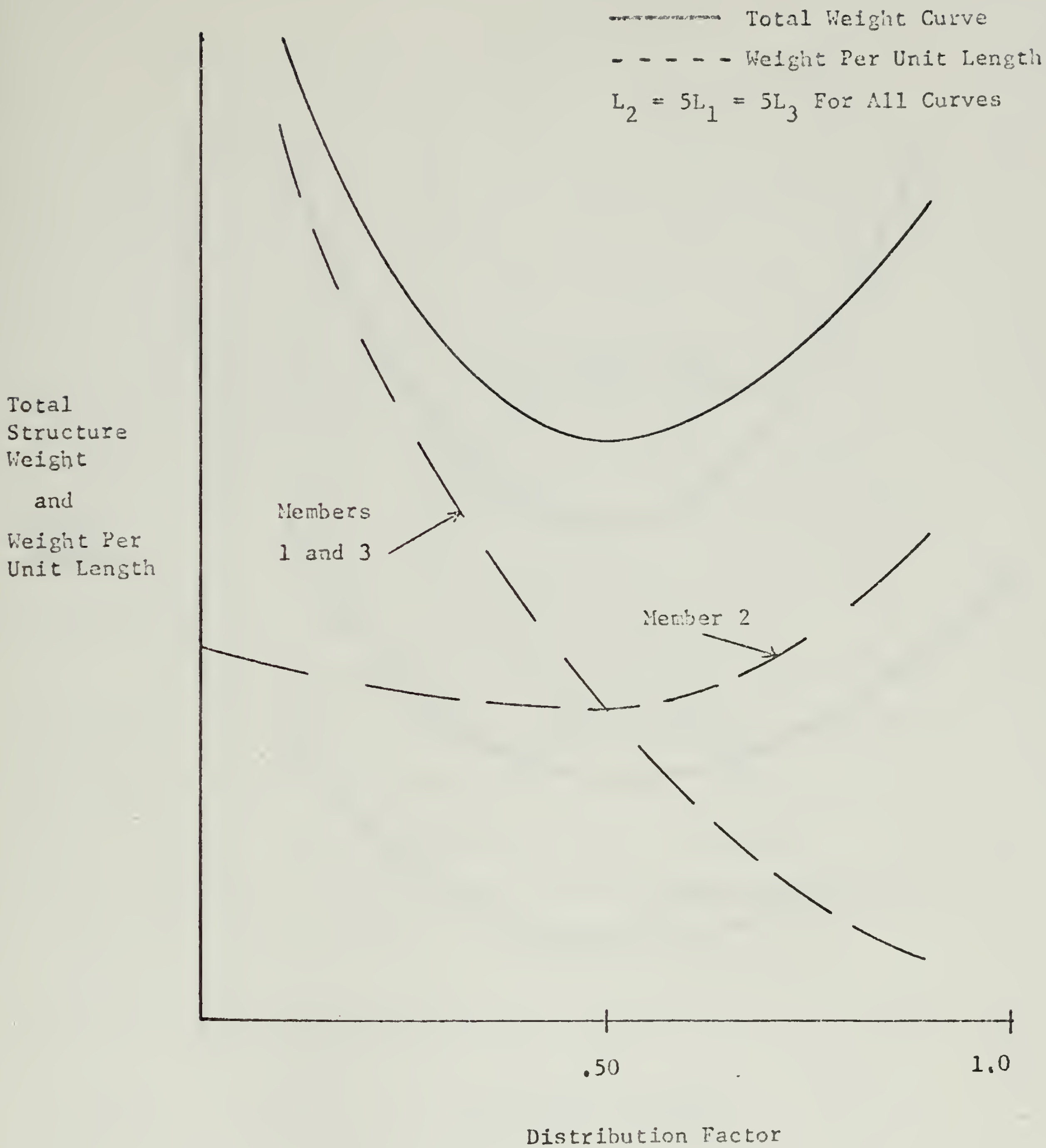
STRUCTURE FOR EXAMPLE 1, SECTION II



STRUCTURE FOR EXAMPLE 2, SECTION II

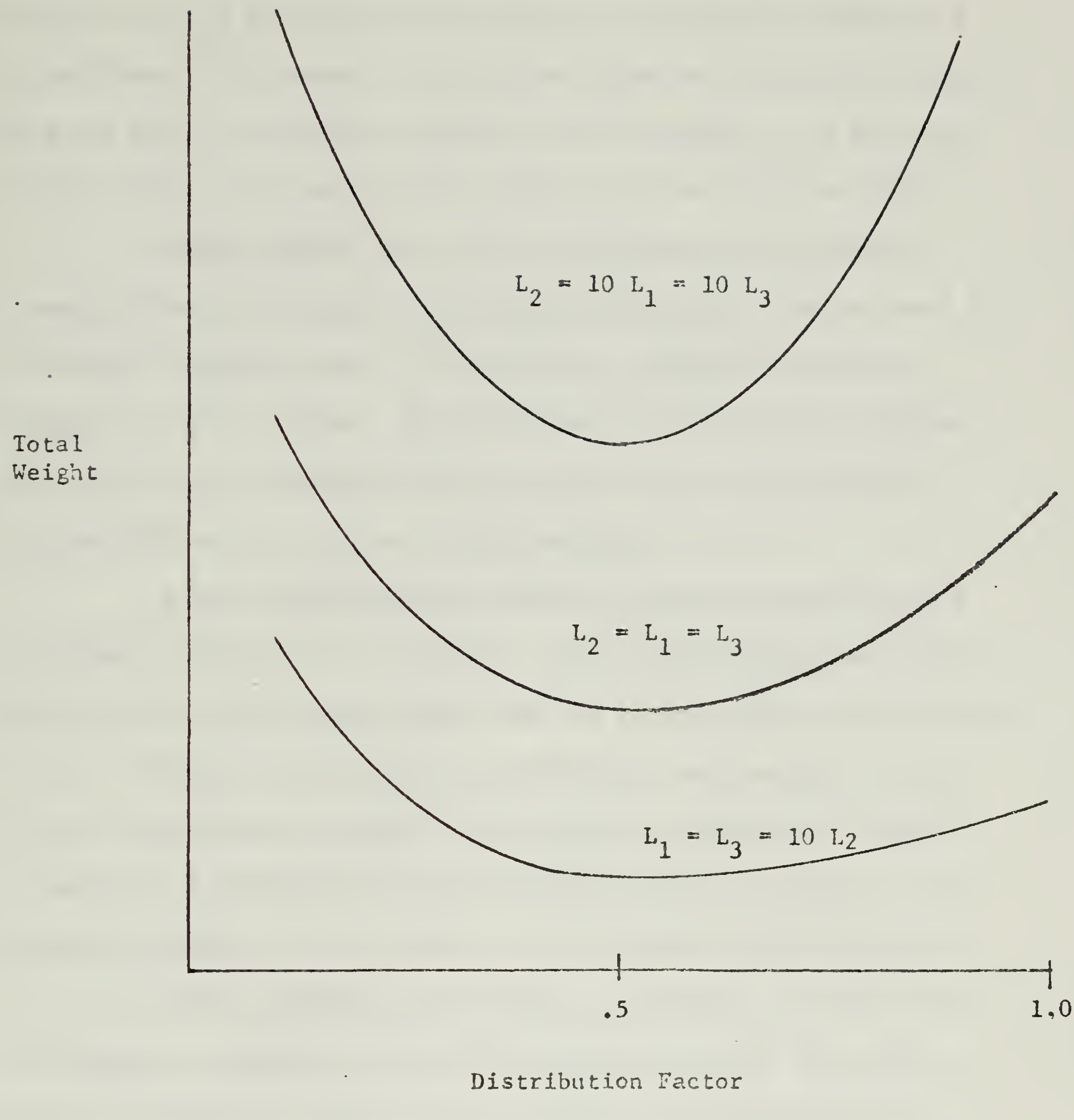
BEAM SYSTEMS FOR EXAMPLES ONE AND TWO

FIGURE 2



THE TOTAL WEIGHT AND WEIGHT PER UNIT LENGTH CURVES FOR EXAMPLE ONE, SECTION II

FIGURE 3



THE TOTAL WEIGHT CURVES FOR EXAMPLE TWO, SECTION II

FIGURE 4

III. CONVERGENCE PROCEDURES

The properties of a two dimensional transverse ring frame of a wall sided ship may be considered similar to those of a plane frame. The members of this plane frame are subjected to bending and direct stresses; side sway is not considered. In its most complex form, this space frame is made up of many bays and decks.

Picking member sizes that are adequate, i.e. possess enough strength to support the applied loads, can be accomplished in a straight forward manner. St. Denis [1], presents a rational approach to this problem. The refinement of these initial choices, i.e. cutting the strength of each member until it just meets the design stress, is discussed in this section.

A two dimensional space frame of more than one deck is a statically indeterminent structure. The stress distribution resulting from the design loads depend upon the size of each member of the frame. Changes in these stress distributions are expected if the size of one member is altered. The effects of changing the size of a member in a reasonably balanced structure will be greatest in the immediate vicinity of the member. It is assumed in the following that the stress or moment distribution in a member is affected only by changes in members that join the defining nodes of the member. This is the same as saying that each member of the frame is the same as member 2 of Figure 5. The stress distribution in member 2 is affected by size changes in any of the members shown.

The refinement problem is this: given a frame with a fixed layout and subjected to specified design loads, what distribution of material will produce stress distributions such that in each member the maximum stress equals the design stress. This means that in each member that has a maximum stress less than design stress, the section is too heavy. Material must be taken away or the stress distribution altered so that the member carries more of the load. If the maximum stress in the member is greater than design stress, the member is undersized and the opposite holds.

An approach to the problem that is suitable to machine applications is a selective procedure. After each stress analysis, the member in the worst condition, over or under design stress, is selected. The size of the member is altered based upon existing maximum moment. The structure with the corrected member undergoes another stress analyses. This cycle is repeated until all members are at the design stress level.

A more appealing approach is one in which changes can be made in all the members before each stress analysis. The results of each stress analysis are used as the basis for changing unsatisfactory members.

The implementation of this series approach requires that interaction effects be estimated. Unless account is taken of how the stress distribution changes when the distribution of material is altered, the system will be subjected to undesirable oscillations.

The member shown in Figure 6 depicts a general member taken from the two dimensional plane frame. It carries a distributed load, W, and has non-symmetrical end restraint. The stress distribution is not symmetrical; node one is assumed to be overstressed.

The information required is how will this moment distribution change when any or all of the members involved change. At each node three conditions have to be considered, an increase, decrease, or no change in the joint rigidity.

An increase in the size of all the members adjacent to node two will provide additional rotational restraint. This is accompanied by an increase in the moment at node two and a decrease in the maximum moment. If the changes in the members adjacent to node two were sufficiently large, no change in member one would be required due to the altered stress distribution.

Increasing the size of member one alone will cause the maximum moment to increase. If the size of the new member is picked based on the existing maximum moment, then it also will be overstressed when the structure is reassembled.

Table 2 is a summary of the conditions that have to be accounted for. Column three has the information needed to make a rational choice of new members. An expected increase in the maximum moment indicates that a larger than usual correction should be made in the already overstressed member. This would be accomplished by adding an increment to the existing moment. The new member size would reflect this added increment.

Additional tables would be required for each of the following: a field moment governing, end moment identical and governing, and non loaded elements.

The Hardy Cross method of moment distribution offers a direct method for obtaining numerical estimates of changes in maximum moments due to shifts in material distribution about nodes. The expression that is derived below actually embodies the tables discussed in the last section.

A first approximateion to the moment M_{12} of Figure 5 can be obtained by carrying out the Hardy Cross method of moment distribution through the first unlocking of all eight joints. It is not necessary to require that the joints three through eight be fixed.

First release the joints adjacent to node two and distribute the unbalanced moments, U. The carry over from joints four, six and eight to node two is:

$$CO = - 1/2\ k_{62}\ U_6 - 1/2\ k_{82}\ U_8 - 1/2\ k_{42}\ U_4$$

This carry over is added to the unbalanced moment at node two, U_2. The amount of this summation that carries over to node one after node two is unlocked is:

$$1/2\ k_{21}\ [1/2\ (k_{62}\ U_6 + k_{82}\ U_8 + k_{42}\ U_4) - U_2]$$

The carry over from the joints adjacent to node one is found in the same way. It is:

$$CO = - 1/2\ k_{51}\ U_5 - 1/2\ k_{71}\ U_7 - 1/2\ k_{31}\ U_3$$

The moment of node one in member 1-2, M_{1-2}, can be expressed in the following approximate form:

$$
\begin{aligned}
M_{12} \approx\; & FEM_{1-2} - k_{12} U_1 + 1/2\, k_{12} (k_{51} U_5 + k_{31} U_3 + k_{71} U_7) \\
& - 1/2\, k_{21} U_2 + 1/2\, k_{12} k_{21} U_2 + 1/4\, k_{21} (k_{62} U_6 + \\
& \qquad k_{82} U_8 + k_{42} U_4) \\
& - 1/4\, k_{21} k_{12} (k_{62} U_6 + k_{82} U_8 + k_{42} U_4) + 1/4\, k_{12} k_{21} U_1 - \\
& \qquad 1/4\, k_{12}^2 k_{21} U_1
\end{aligned}
\tag{1}
$$

If M_{12} is the maximum moment in the member, the change in the moment as the adjoining members and member two itself changes is desired. The quantities FEM_{ij} and U_i depend only on the loads and the geometry and therefore are constants. The expression for M_{12} can be looked upon as a function of the distribution factors, k_{ij}, which in turn depend directly on the section properties.

$$M_{12} \approx f\,(k_{12} \ldots\ldots\ldots k_{ij})$$

The change in M_{12} is

$$dM_{12} = \frac{\partial f}{\partial k_{12}}\, dk_{12} \ldots\ldots + \frac{\partial f}{\partial k_{ij}}\, dk_{ij}$$

or

$$\Delta M_{12} = \frac{\partial f}{\partial k_{12}}\, \Delta k_{12} \ldots\ldots + \frac{\partial f}{\partial k_{ij}}\, \Delta k_{ij}$$

Applying this to equation (1) results in:

$$\Delta M_{12} \simeq - U_1 \Delta k_{12} + 1/2 (k_{51} U_5 + k_{31} U_3 + k_{71} U_7) \Delta k_{12}$$

$$+ 1/2 k_{12} (\Delta k_{51} U_5 + \Delta k_{31} U_3 + \Delta k_{71} U_7) - 1/2 U_2 \Delta k_{21}$$

$$+ 1/2 U_2 (k_{12} \Delta k_{21} + \Delta k_{12} k_{21}) + 1/4 (k_{62} U_6 + k_{82} U_8 +$$

$$k_{42} U_4) \Delta k_{21}$$

$$+ 1/4 k_{21} (\Delta k_{62} U_6 + \Delta k_{82} U_8 + \Delta k_{42} U_4) + 1/4 U_4$$

$$(k_{12} \Delta k_{21} + \Delta k_{12} k_{21})$$

(2)

An adequate comparison of the orders of magnitude of the different parts of this expression requires knowledge of the joint unbalance. If it is assumed, as it was previously, that the loads are reasonably balanced throughout the structure, the structure of Figure 5 is again applicable. Applied to this structure equation (2) reduces to:

$$\Delta M_{12} \simeq - U_1 \Delta k_{12} - 1/2 U_2 \Delta k_{21} + 1/2 U_2 (k_{12} \Delta k_{21} + \Delta k_{12} k_{21})$$

$$+ 1/4 U_1 (k_{12} \Delta k_{21} + \Delta k_{12} k_{21})$$

(3)

This is the same as neglecting the unbalance at joints three through eight. The inclusion of these terms in a computer program is straightforward, but of course, would require more computational time. This final approximateion accounts for changes in any of the seven members involved. A more convenient form for equation (3) is:

$$\Delta M_{ij} = [-U_i \ (1 - 1/4 \ k_{ji}) + 1/2 \ k_{ji} \ U_j] \ \Delta k_{ij}$$

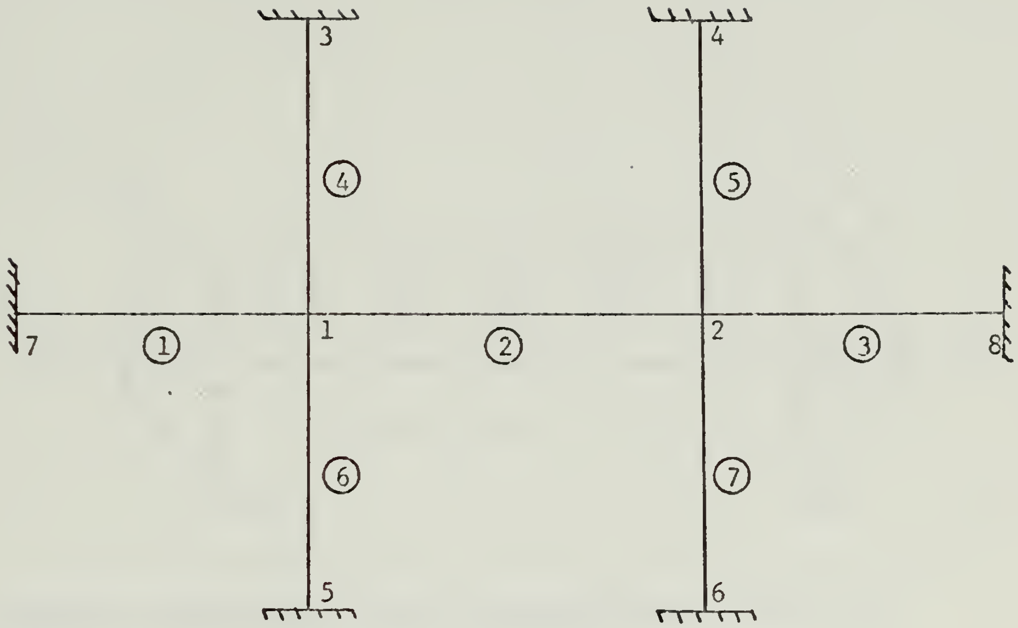
$$+ 1/2 \ [- U_j \ (1 - k_{ij}) + 1/2 \ U_i \ k_{ij}] \ \Delta k_{ji} \qquad (4)$$

Equation (4) gives a numerical value for the expected change in M_{ij} if the adjoining members and member itself change size. This is an approximation as to how the moment distribution in member ij is affected when any or all the members change size. This is a valid approximation for both loaded and unloaded elements. There is no restriction to members undergoing only bending stress; it is equally useful for members with combination stresses. If different types of materials are used, their effects are included in the distribution factors.

The use of this expression to find changes in the field moment is a little more involved. Equation (4) has to be evaluated for each end of the member in question if the moment distribution

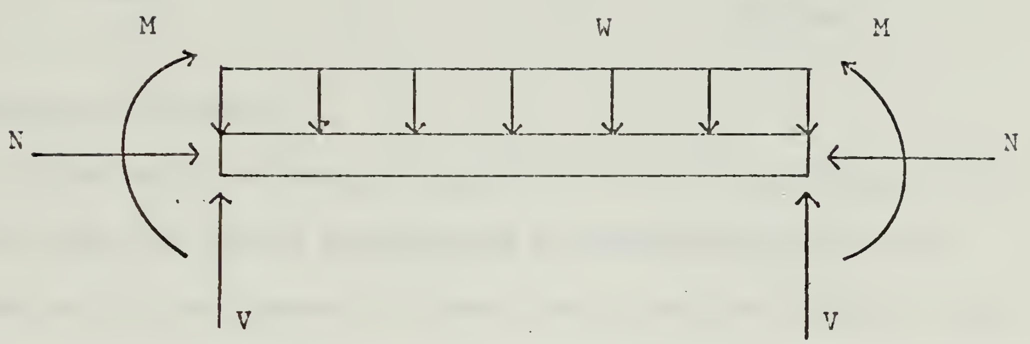
is not symmetrical and added to the existing end moments. These new moments combined with the existing end shears give an approximation to the change in field moment.

A word about signs is appropriate. Clockwise moments are positive, counterclockwise moments are taken to be negative. The signs that result when ΔM_{ij} is calculated cannot be interpreted literally. A positive value for ΔM_{ij} may indicate an expected decrease in the moment, M_{ij}. It is necessary to check the sign of the existing moment and then interpret the sign of ΔM_{ij}.



A SUBSECTION OF THE TWO-DIMENSIONAL RING FRAME

FIGURE 5



A GENERAL BEAM MEMBER OF THE TWO-DIMENSIONAL RING FRAME

FIGURE 6

TABLE 2

Cumulative Effects of Member Variations

	(1) Adjacent Members	(2) Member to be Changed	(3) Exp. Change in Max. Mom.
End w/ Mom)$_{Min}$	-	+	Increase
	+	+	No Change
	o	+	Increase
End w/ Mom)$_{Max}$	-	+	Decrease
	+	+	No Change
	o	+	Decrease

Explanation of Table

This table is drawn up for Member Two of Figure Five. It is assumed that the moment distribution is nonsymmetrical and that Member Two is overstressed. Column three gives the expected change in the maximum moment based upon the cumulative effects of member changes summarized in columns one and two.

Column 1: The sign indicates how the overall stiffness of the joint is expected to change based upon the stress conditions in the adjacent members.

Column 2: The size of Member Two is expected to increase

Column 3: The expected change in the maximum moment based upon the data of columns one and two.

IV. THE DESIGN SPIRAL

The result of the previous section is a technique for estimating moment changes. These moment changes are brought about when the members at a node change their size.

This is of use in the design process because it presents a method of estimating not only the magnitude of changes in the stress distribution (maximum moment), but also the direction of the changes. It permits a more rapid convergence to the final structure because each member of the structure can be modified (if necessary) on each pass. And also because the members are picked based upon conditions that will prevail; not those of the last stress analysis. It is essentially a method of predicting the final moment distribution and picking the members based on this distribution. If the moment changed predicted were exact instead of approximations there wouldn't be any iterative procedure necessary.

A desirable aspect of this technique is that it is self modifying when applied repeatedly. The predicted moment changes are updated each time a new member is picked. In this way oscillations are avoided and smooth convergence is assured. It is not necessary to perform a new structural analysis until this self modifying aspect is completed. This occurs when there are no

significant differences in the predicted moment changes from one cycle to the next.

The rational design process developed in the sections that follow converges to the optimum distribution of material for a transverse ring frame. The general procedure followed is to use the results of a stress analysis as the basis for modifying the material distribution in such a manner that design stresses will be attained in each member. The results of Section II indicate that a structure which develops design stresses in each member simultaneously is the least weight structure.

Equation (4) is used in this process to make corrections to the results of the stress analysis. A change in the scantlings of any member of the ring frame changes the stress distributions. As pointed out in Section III and summarized in Table II, the effect of a change in scantlings depends not only on the magnitude of the change, but also the existing moment distribution. Once the maximum moment in the member is identified, equation (4) gives direct information as to how this moment will change when the scantlings of the members of the ring frame are revised.

This correction to the existing moment distributions provides the means of facilitating rapid convergence to the optimum structure. Changes in member scantlings are based on these predicted moments. The objective of design stress in each member of the ring frame is obtained much more readily.

The use of equation (4) in the design process requires the standard output of a stress analysis, end forces, plus end moments. The process begins with the results of the stress analysis which used the initial choices of members.

The computer begins with member one. The governing moment is ascertained and the stress level is computed based on this maximum moment. If the stress level is within a certain prescribed range of the design stress, the member is not changed. This provides a means for ending the iterations. If the stress level is not satisfactory, a new member size is calculated and stored in the matrix, K. This procedure is repeated for each member of the structure. The result of this cycle is the matrix K. The members that make up K represent the first cut at bringing about the desired distribution of material. These members are fictitious in the sense that they are not used in an actual stress analysis. The properties of these members are based upon the existing stress distributions.

The computer now starts to revise the members of matrix K; it starts with the first loaded member. The stiffnesses, k_{ij}, of the members at the defining nodes of the member to be changed are computed using the data of matrix K. This data is used to compute ΔM_{ij} or ΔM_{ji} or both depending on the stress distribution. If the field moment governs ΔM_{ij} is computed for each node; when an end moment governs ΔM_{ij} is calculated for the governing node.

The maximum moment of the member to be changed is modified by adding the correction factor ΔM_{ij}. The properties of a new section are calculated based on this modified moment. These properties are inserted in the proper slot of matrix $\bar{K}$.

The computer chooses the next loaded member and repeats the above. The unloaded members are revised after new properties have been picked for all the loaded members. This requires fewer steps because the end moments always govern.

The result after all the members have been looked at is a new matrix of members, K'. A stress analysis could be initiated at this point. The results of the examples indicate, however, that a smoother convergence is obtained if this process is repeated, i.e. find the matrix K''.

A stress analysis is now undertaken using the members of the matrix K''. The results of this stress analysis are used as the starting point for a new design cycle.

Flow charts containing the details of the design spiral are contained in Appendix A. Illustrations of the design spiral in action are contained in Appendix B.

V. RESULTS AND CONCLUSIONS

A rational design procedure has been developed which permits the rapid synthesis of the transverse ring frame of a wall sided ship. The resulting transverse structure represents the optimum distribution of material, the minimum wieght combustion of deck beams, side frames, etc., within the constraints of fixed longitudinal and transverse frame spacing.

Detailed applications of this procedure are carried out in Appendix B. Each step follows the subroutines developed for computer adaptation of this technique. No attempt has been made to adapt the procedure to hand calculations.

This method converges rapidly. Large member changes were required in each of the examples to attain the design stress in each member. Example I required only one additional stress analysis after the initial choice of members were analyzed; Example II required two additional moment distributions.

The speed with which a satisfactory ring frame is obtained depends directly on the following: the limits set within which the member stress must fall and the use of weighting functions in conjunction with the predicted moment changes.

The limits set for member stresses in Appendix B were liberal. This, of course, was conducive to a rapid convergence.

These limits are not fixed, however, and can be set at any desired level. It is possible to put new limits on the members stress with each problem.

The predicted moment changes in the examples were used as given by equation (4). The true approximation in equation (4) is in the magnitude of expected moment change. The direction of the change, plus or minus, is exact if the change is greater than a few percent. In all the cases investigated, the magnitude of the change was always low. Convergence would have been more rapid if the changes were multiplied by a weighting factor. This indicates that weighting factors, if necessary, can be developed as experience is gained with the basic program. These weighting factors would become part of the basic program.

The results of Example II of Appendix B indicates that members whose continuity is not required to carry the applied loads may be eliminated. This is easily avoided by applying nominal loads to these members, e.g. their own weight. In an actual structure, this will seldom be a problem.

The use of the ideal section of Appendix C was a necessity in the development stages of this program. The implementation of this design procedure, i.e. the actual programming, does not require this ideal section. If a tape of actual sections is available, the program can use it and still work. The effect this would have

on the speed of convergence would be mixed. The actual members in most cases have larger section moduli than those the program would pick. The use of actual sections requires, in addition, the development of a trial and error solution in the subroutine CONSTRESS.

APPENDIX A

An algorithm covering the complete design process is developed. Flow charts covering the details of the individual steps of the algorithm are also presented.

Design Algorithm

Start with member one compare the maximum and design stresses. If the maximum stress is within the prescribed range, insert the present member properties in matrix K. If maximum stress unsat., pick a new member. Insert its properties in the matrix K.

Repeat the above for each member. If all members have satisfactory stress levels the computer stops - the design is completed. If any or all members were changed, the iteration process is begun by returning to the loaded member number one.

Required stiffness factors are calculated using elements in matrix K. The moment change is determined using this data. The existing maximum moment plus this moment change is the new design moment. A new member is picked using this design moment and the direct stress.

The new member is placed in matrix K. This process is repeated for all members even if stress levels are initially satisfactory. The matrix containing these new members is now designated K'.

The data of matrix K' is now used to calculate the required stiffness factors. The process is repeated. The matrix formed is K''.

A stress analysis is carried out using matrix K''. The results of this analysis are the starting point for a new design cycle.

The flow charts are made up in the form of subroutines. They cover all the steps presented in the design algorithm.

The first flow chart is subroutine Moment. It uses as an input the data of the preceding stress analysis. This subroutine primarily finds the maximum moment and calculates the maximum stress. When field moments govern the member is given a special tag.

Subroutine Change checks the maximum stress in each member. It stops the iteration process if all members are satisfactory.

Subroutine Stiffness uses the data of matrix K and the data of initial member choices to calculate the expected changes in stiffness. It also determines the moment change and adds this to the maximum moment. The new member parameters are obtained by calling the appropriate subroutine, i.e. Pick or Comstress. This subroutine assumes that a member numbering system is available which facilitates picking the required data from the matrix K, etc.

Subroutines Pick and Comstress both choose new member properties. Comstress handles members that have significant direct stress as well as bending stresses.

The calling sequence for these subroutines would be:

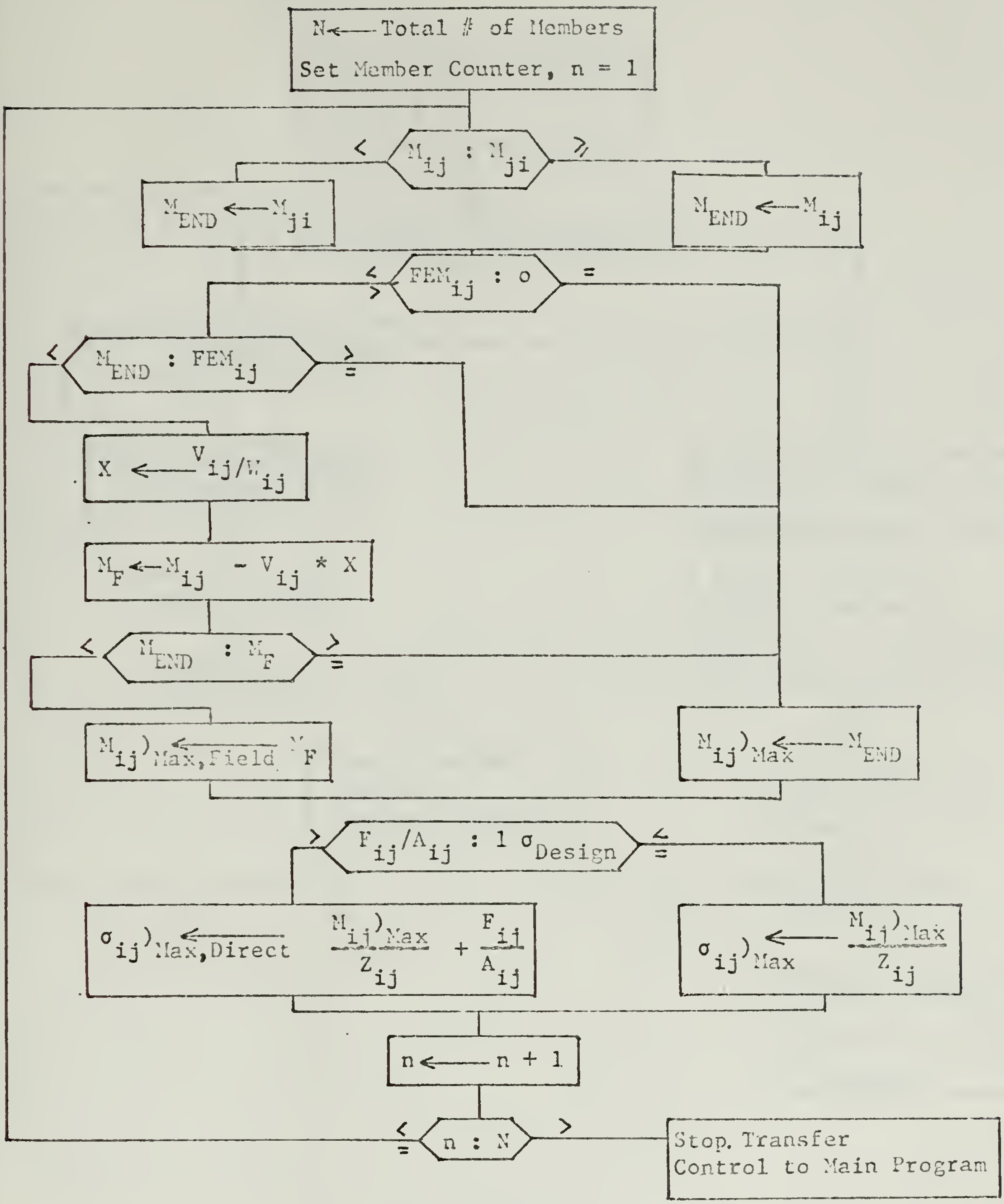
CALL SUBROUTINE MOMENT

CALL SUBROUTINE CHANGE

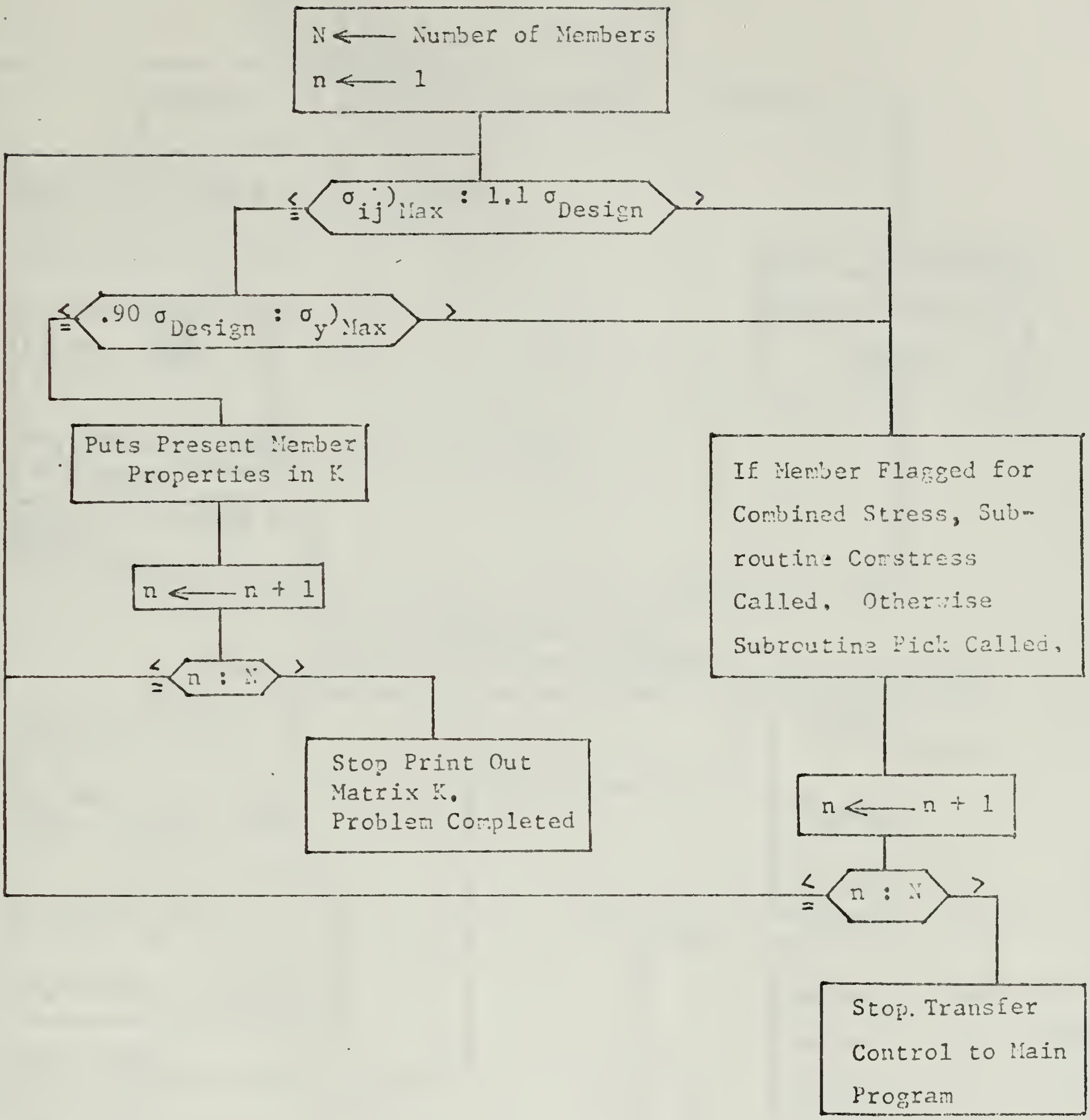
CALL SUBROUTINE STIFFNESS

CALL STRESS - or any suitable stress analysis program.

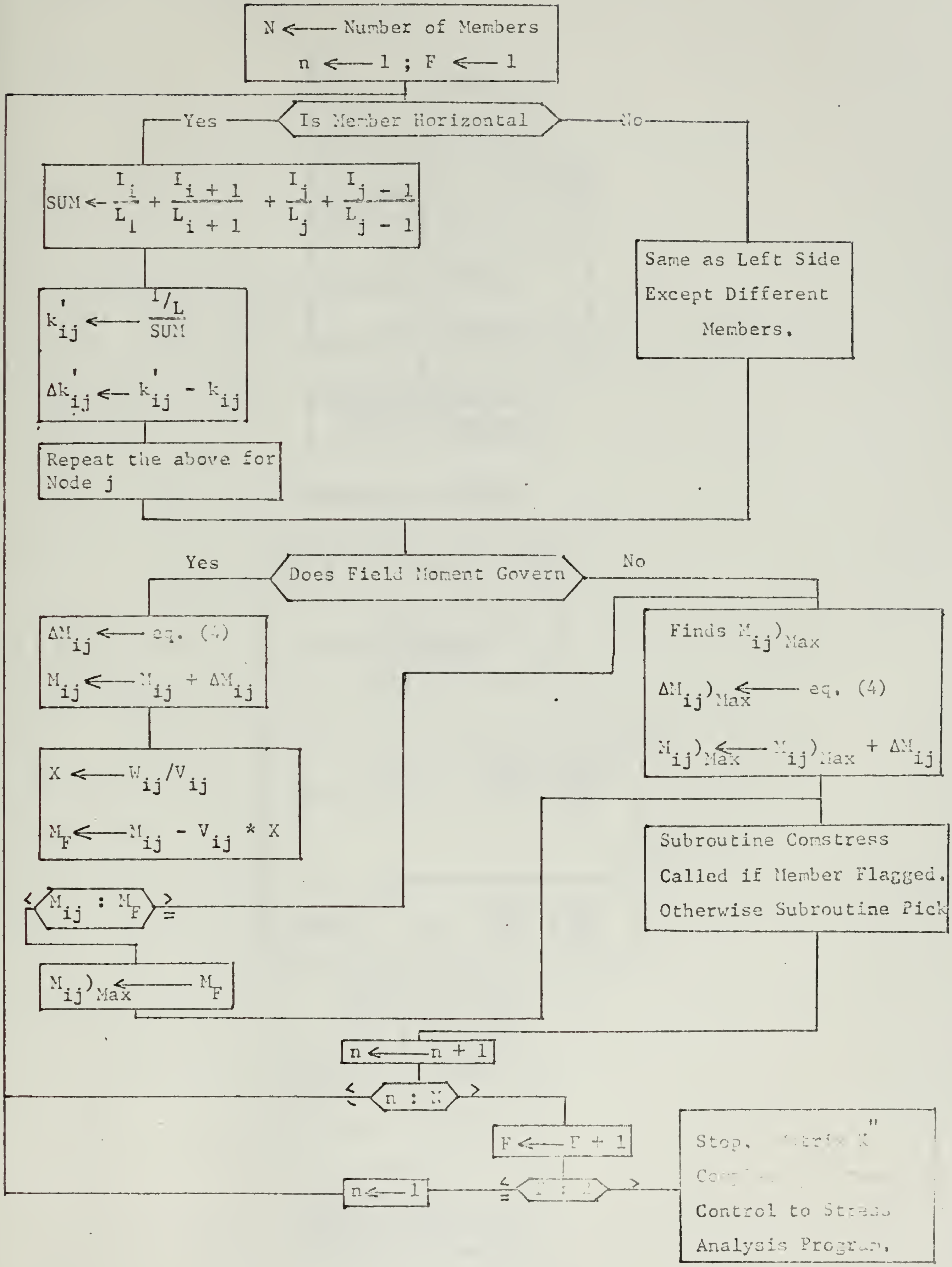
SUBROUTINE MOMENT



SUBROUTINE CHANGE



SUBROUTINE STIFFNESS



SUBROUTINE PICK

$$Z_{ij} \longleftarrow \frac{M_{ij})_{Max}}{\sigma_{Design}}$$

$$D_{ij} \longleftarrow (Z_{ij}/.032)^{1/3}$$

$$A_{ij} \longleftarrow (.052)\ D^2$$

$$I_{ij} \longleftarrow (.0154)\ D^4$$

Return Control to Calling Program

SUBROUTINE COMSTRESS

$$a \longleftarrow -\ F_{ij}/\sigma_o \times .052$$

$$b \longleftarrow \frac{M_{ij})_{Max} \times 12}{\sigma_o \times .032}$$

$$A \longleftarrow \sqrt[3]{-\ b/_2 + \sqrt{\frac{b^2}{4} + \frac{a^3}{27}}}$$

$$B \longleftarrow \sqrt[3]{-\ b/_2 - \sqrt{\frac{b^2}{4} + \frac{a^3}{27}}}$$

$$D \longleftarrow A + B$$

$$A_{ij} \longleftarrow .052\ D^2$$

$$Z_{ij} \longleftarrow .032\ D^3$$

$$I_{ij} \longleftarrow .0154\ D^4$$

Transfer Control to Calling Subroutine

APPENDIX B

EXAMPLES

EXAMPLE I

The seven membered frame shown on page 45 is used to illustrate the optimization technique developed in the body of the thesis.

A. Selection of Initial Member Properties

The initial set of member properties is part of the data input to the computer. The following were selected for this example.

(a) $Z_2 = 240 \text{ in}^3$; $I_2 = 2000 \text{ in}^4$

(b) $Z_1 = Z_3 = 120 \text{ in}^3$; $I_1 = I_3 = 820 \text{ in}^4$

(c) $Z_4 = Z_5 = Z_6 = Z_7 = 120 \text{ in}^3$; $I_4 = 820 \text{ in}^4$

These properties were obtained using the ideal section of Appendix C.

B. Stress Analysis

The Hardy Cross method of moment distribution was used to obtain end moments and reactions. The results of this first analysis are:

$M_{12} = M_{21} = M_1)_{Max} = 684$ KIP FT

$M_{17} = M_{28} = M_2)_{Max} = 495$ KIP FT

$M_{13} = M_4)_{Max} = 95$ K'; $F_4 = 180$ KIP FT

C. Subroutine Change

(1) The maximum stress in each member is compared to the design stress = 40 KSI

(2) The size of each unsatisfactory member is changed.

The maximum stress in each member is:

$\sigma_1)_{Max} = 34$ KSI

$\sigma_7)_{Max} = 50$ KSI

$\sigma_3)_{Max} = 15$ KSI + 10 KSI = 25 KSI - a combined stress member.

The new properties for each member are:

$$Z_2 = \frac{M_2)_{Max}}{\sigma_{Design}} = 204 \text{ in}^3; \; I_2 = 1600 \text{ in}^4$$

$$Z_7 = 148 \text{ in}^3; \; I_7 = 1100 \text{ in}^4$$

$$Z_4 = 67 \text{ in}^3; \; A_4 = 8.3 \text{ in}^2; \; I_4 = 415 \text{ in}^4$$

The remaining member sizes follow by symmetry. These properties make up the matrix K. The subroutine PICK would be used to obtain the properties of members one through three; subroutine COMSTRESS would be called to obtain the properties for members four through eight.

D. Subroutine Stiffness

(1) Calculates new stiffness factors, k'_{ij}

(2) Calculates the change in stiffness factors with respect to the initial member choices.

(3) Determines ΔM_{ij} and the new factitious maximum moment

(4) Picks new members based one the new maximum moment.

This subroutine begins with the first loaded member, member 1 and calculates k'_{17}

$$k'_{17} = .310 \qquad k'_{71} = 0$$

$$k_{17} = .184 \qquad k_{71} = 0$$

$$\Delta k'_{17} = + .126 \qquad \Delta k_{71} = 0$$

These results are used in equation (4) to obtain ΔM_7,

$$\Delta M_{1-7} = + 50.4\ K' \ ; \ M_{1-7} = + 495 + 50.4 = 545 \text{ KIP FT}$$

Subroutine Pick now comes up with

$$Z'_7 = 153 \text{ in}^3; \ I'_7 = 1140 \text{ in}^4$$

These member properties are immediately inserted in the matrix K. This provides a constant updating of the data.

The computer proceeds thru the structure doing the above for each member

Member 2

$$k'_{12} = .448 \qquad k'_{21} = .452$$

$$k_{12} = .450 \qquad k_{21} = .450$$

$$\Delta k'_{12} = -.002 \qquad \Delta k'_{21} = + .002$$

$$\Delta M'_{1-2} = 1.1K'; \ - M'_{12} = 685 \text{ KIP FT}$$

$$Z'_2 = 204 \text{ in}^3; \ I'_2 = 1600 \text{ in}^4$$

D. Subroutine Stiffness (Cont'd)

Member 3

$k'_{17} = .310$ $k'_{71} = 0$

$k_{17} = .184$ $k_{71} = 0$

$\Delta k_{17} = + .126$ $\Delta k_{71} = 0$

$\Delta M'_7 = + 50.4$ KIP FT $M'_7 = 545$ KIP FT

$Z'_7 = 153 \text{ in}^3$; $I'_7 = 1140 \text{ in}^4$

Member 4

$k'_{13} = .117$ $k'_{31} = 0$

$k_{13} = .184$ $k_{31} = 0$

$\Delta k'_{13} - .067$ $\Delta k'_{31} = 0$

$\Delta M'_3 = 27$ KIP FT $M'_3 = 73$ KIP FT

$Z'_3 = 55 \text{ in}^3$; $I'_3 = 310 \text{ in}^4$ Subroutine COMSTRESS

Member 5

$k'_{15} = .121$ $k'_{51} = 0$

$\Delta k_{15} = .184$ $\Delta k_{51} = 0$

$\Delta k'_{15} = -.063$ $k_{51} = 0$

$\Delta M'_5 = -25.2$ KIP FT $M'_5 = 75$ KIP FT

$Z'_5 = 55 \text{ in}^3$, $A'_5 = 7.2 \text{ in}^2$; $I'_5 = 310 \text{ in}^4$

Members 6 and 7 follow by symmetry.

The matrix K at this point becomes the matrix K'. Subroutine Stiffness repeats the above process and produces matrix K''. The results of this procedure are summarized below

Member	Z' (in^3)	A' (in^2)	I' (in^4)
1	167	----	1300
2	204	----	1600
4	50	6.9	280
5	50	6.9	280

The matrix K'' is now the matrix of actual member properties. A stress analysis of the structure made up of these members is performed. The results of this analysis are:

$M_1 = 595$ KIP FT $\quad \sigma_1 = 42$ KSI $= 1.07\ \sigma_o$

$M_2 = 680$ KIP FT $\quad \sigma_2 = 40$ KSI $= \sigma_o$

$M_4 = .41$ KIP FT $\quad \sigma_4 = 36$ KSI $= .9\sigma_o$

Stresses and moments for other members follow by symmetry.

Subroutine Change now finds all the stress levels satisfactory. The members of matrix K'' represent the optimum structure. The design spiral is complete.

EXAMPLE 2

This example was chosen to illustrate the procedure for a nonsymmetrical system and one in which the field moment governs, at least initially.

The frame analyzed is shown on page 46 . The optimization process proceeds as illustrated in detail in example one.

A. Initial Member Properties

$Z_2 = 240 \text{ in}^3$; $I = 2000 \text{ in}^4$

All other members - $Z = 40 \text{ in}^3$; $A = 6 \text{ in}^2$; $I = 210 \text{ in}^4$

B. Stress Analysis

$M_1 = 168$ KIP FT

$M_2 = 780$ KIP FT

$M_4 = 168$ KIP FT

$M_5 = 84$ KIP FT

C. Subroutine Change

$\sigma_1 = 50$ KSI

$\sigma_2 = 39$ KSI ; M_F governing

$\sigma_4 = 50$ KSI

$\sigma_5 = 38.5$ KSI

Combined Stress

$Z_1' = 50 \text{ in}^3$; $I_1' = 290 \text{ in}^4$

$Z_2' = 240 \text{ in}^3$; $I_2 = 200 \text{ in}^4$

$Z_4' = 83 \text{ in}^3$; $A_4' = 9.5 \text{ in}^2$; $I_4' = 560 \text{ in}^4$

$Z_5' = 37 \text{ in}^3$; $A_5' = 5.6 \text{ in}^2$; $I_5' = 190 \text{ in}^4$

D. Subroutine Stiffness

Matrix K'

Member	Z' (in^3)	A' (in^2)	I' (in^4)
1	56.1	----	320
2	208	----	1700
4	140	13.5	1050
5	23	4.1	100

The field moment governed in member 2 at this point. However once the second set of members were obtained, the end moments predominated. The results of the second pass is matrix K''

Member	Z'' (in^3)	A'' (in^2)	I'' (in^4)
1	56.1	----	320
2	203	----	1650
4	190	16.5	1500
5	15	3	60

E. Stress Analysis

M_1 = 530 KIP FT

M_2 = 655 KIP FT

M_4 = 114 KIP FT

M_5 = 0

F. Subroutine Change

σ_1 = 38.8 KSI = .97 σ_o

σ_2 = 30 KSI = .75 σ_o

σ_4 = 48 KSI = 1.25 σ_o

σ_5 = 0

These stress values require another pass. The trend is obvious. Members two, four and six are taking all the load. The second iteration with subroutine Stiffness produces the following matrices.

Matrix K' - second pass

Member	Z' (in^3)	A' (in^2)	I' (in^4)
1	17	----	40
2	203	----	1650
4	262	21	2000
5	----	----	----

Matrix K'' - second pass

Member	Z'' (in^3)	A'' (in^2)	I'' (in^4)
1	----	----	----
2	192	----	1550
4	262	21	2000
5	----	----	----

G. Stress Analysis

M_2 = 646 KIP FT

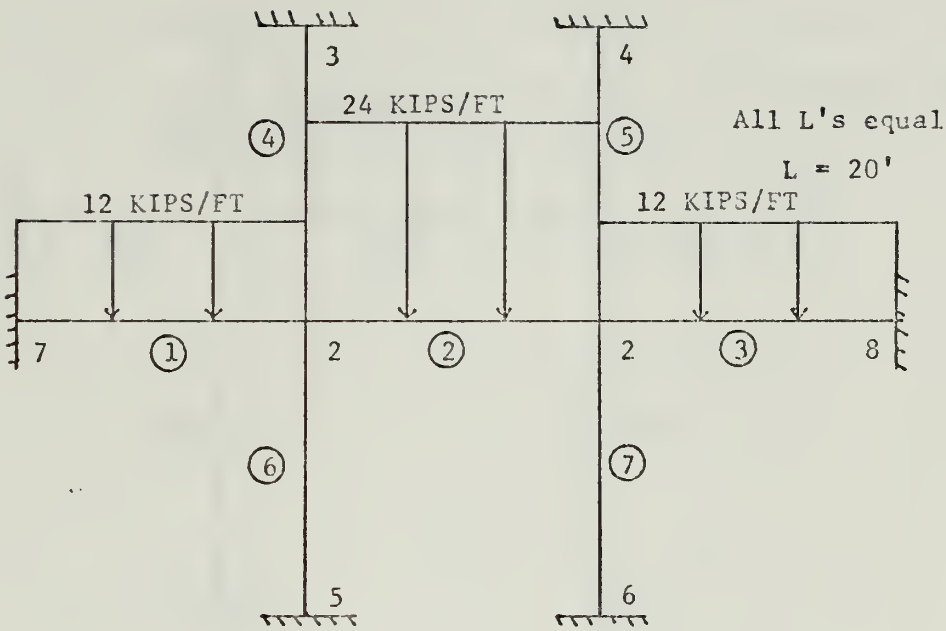
M_4 = 646 KIP FT

H. Subroutine Change

σ_2 = 39.5 KSI = .98 σ_o

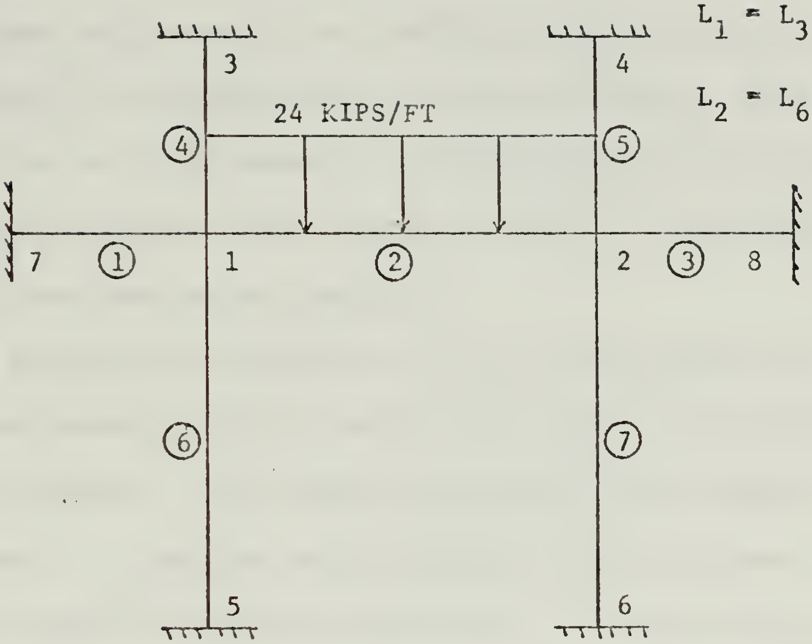
σ_4 = 29.6 + 11.1 = 40.7 KSI = 1.025 σ_o

These stresses are within the specified limits. The design spiral is terminated. The members of matrix K'', second pass, form the optimum structure.



STRUCTURE FOR EXAMPLE B-1

FIGURE B-1



$L_1 = L_3 = L_4 = L_5 = 10'$

$L_2 = L_6 = L_7 = 20'$

STRUCTURE FOR EXAMPLE B-2

FIGURE B-2

APPENDIX C

THE IDEAL SECTION

The section moduli of available steel sections listed in the Manual of Steel Construction do not vary in a continuous manner. The use of the least weight sections compounds the problem. Each section is useful for a wide range of section moduli.

These facts make it very difficult, if one is using the sections listed in the AISC Manual, to pick out trends. Also the fact that there are no mathematical relationships existing between the important properties of the sections compounds the problems encountered when using actual sections.

A mathematical section, i.e. one in which there are specific relationships existing between the section parameters, avoids these problems. The section developed for this thesis is outlined below. It conforms well to minimum weight sections listed in the AISC Manual for larger section moduli. At the lower end of the scale there is a greater divergence from actual sections.

Let t_f = flange thickness

t_w = web thickness

D = depth of section

w_f = flange width

The relationships chosen for the section are:

$$t_f = 2\ t_w$$

$$D = 50\ t_w$$

$$w_f = 20\ t_w$$

The expressions for the area, section modulus and moment of inertia are:

$$A = .052\ D^2$$

$$Z = .032\ D^3$$

$$I = .0154\ D^4$$

A log-log plot of these expressions is made on page 49.

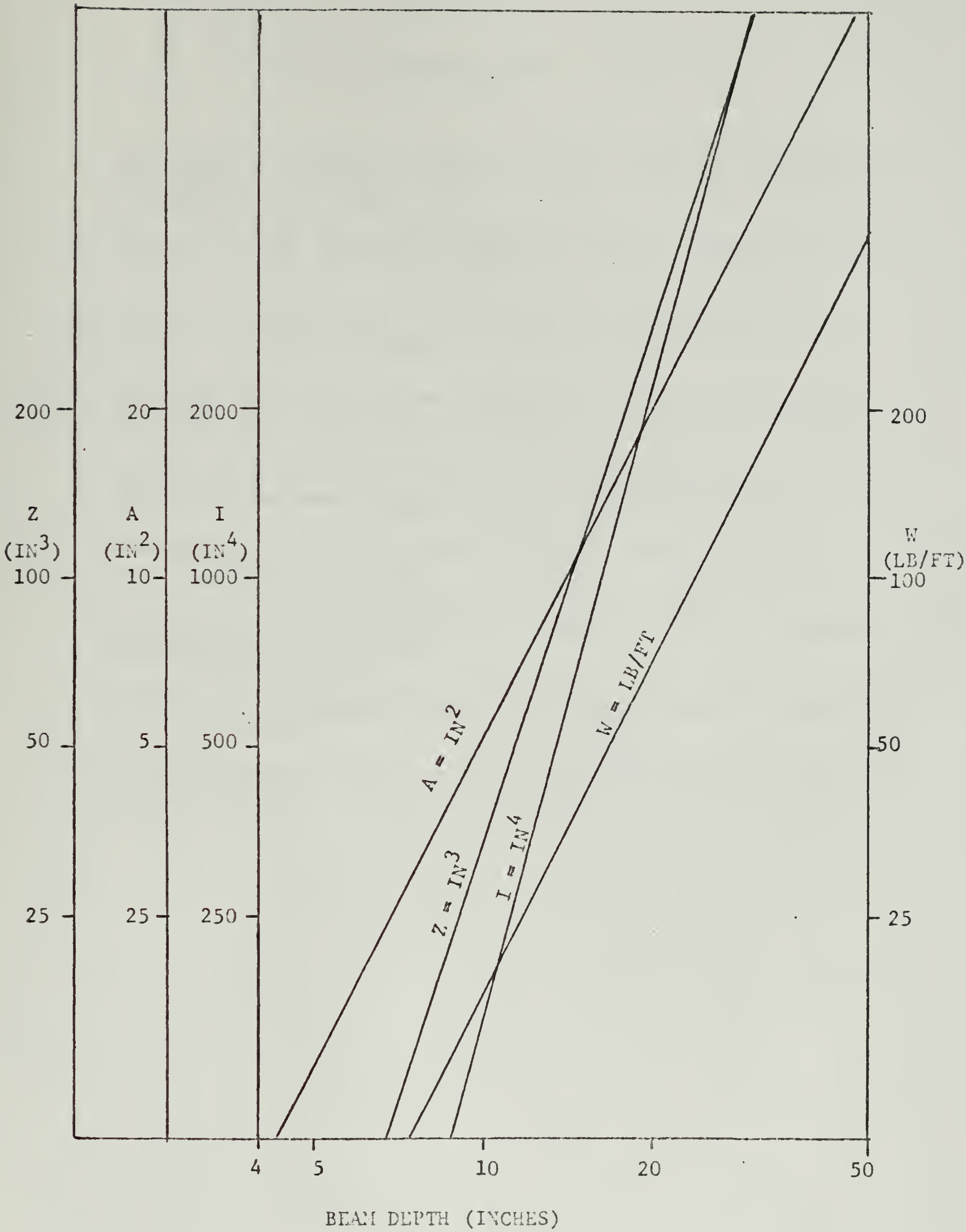

200
20
2000
Z
(IN^3)
A
(IN^2)
I
(IN^4)
100
10
1000
50
5
500
25
25
250
200
W
(LB/FT)
100
50
25
A = IN^2
Z = IN^3
I = IN^4
W = LB/FT
4
5
10
20
50
BEAM DEPTH (INCHES)

FIGURE C-1

APPENDIX D

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